Topic Exploration Pack

# Exponential Growth and Decay

# **Activity 1**

You are investing some money (£100) into a compound interest account but you don’t know which one to choose as there are a number of different banks offering different deals:

Bank 1: Interest rate 50% over each ½ year

Bank 2: Interest rate 25% over each ¼ year

Bank 3: Interest rate 12.5% over each 1/8 year

Bank 4: Interest rate 6.25% over each 1/16 year

After 1 year which one provides the most interest on your original investment of £100?

What happens if you continue the pattern of halving the interest rate and the doubling the number of times you receive interest?

The following tables will help you with the first couple of examples.

1. Complete the tables for Banks 1 and 2:

| **BANK 1 – 2 period 50% each** | |
| --- | --- |
| Time Period | Amount |
| 1 |  |
| 2 |  |

| **BANK 2 – 4 period 25% each** | |
| --- | --- |
| Time Period | Amount |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

1. Find the amounts at the end for banks 3 and 4 by drawing a table or a direct calculation.
2. What happens if we continue the process of halving the interest and doubling the times it is paid? Continue the process three more times.
3. Create a spreadsheet that will calculate what will happen if this process is continued say 100 times. What happens?
4. Divide the final amount by 100 and type this number into an internet search engine. What do you find? Research this special number.

### Activity B

Newton’s law of cooling states that the temperature decreases at a rate proportional to the difference of the temperature from the room temperature. You are going to predict how an object cools and compare it with the actual temperatures.

1. Write down the initial difference in temperature between the object and room temperature in the table below:

| **Time (minutes)** | **Difference in Temp between object and the room** |
| --- | --- |
| 0 |  |
| 1 |  |
| 2 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |

1. Write down the difference in temperature between the temperature taken after 1 minute and the room temperature.
2. Work out the percentage multiplier.
3. Now complete the rest of the table up to 10 minutes.
4. What will the temperature be at the end of the lesson?
5. How long will it take to reach room temperature?
6. Create a graph with the time on the x axis and the difference from room temperature on the y axis.
7. On the same axes plot the actual results. How far out were your predictions?

### Activity C

There are initially 10 rabbits in a field. A number of models are proposed to model their growth which you are going to investigate.

**Model 1**

The rabbit population grows exponentially at a fixed rate of multiplying by a factor of 1.46 each year.

1. Find the population at the end of each year for the next 10 years
2. Write down the equation for the number of rabbits *R* after *n* years
3. How many years does it take for there to be a million rabbits in the field?
4. What is wrong with this model?

**Model 2**

The rabbit population grows exponentially according to the rule:

*R* = 400 ­– 390 x 1.01*n*

1. Find the population after 10 years, 20 years, 30 years, 40 years and 50 years. by using a calculator and substituting the values in.
2. How many years does it take for there to be a million rabbits in the fields?
3. What happens in this model? By plotting the graph of R against n in multiples of 10 what can you say happens to the population in the long term?

### Activity D

The half-life of a radioactive substance is the time taken for a given amount of the substance to become reduced by half because of radioactive decay.  
There are two definitions of half-life, but they mean the same in practice.   
Half-life is the time taken for:

* The number of atoms in a sample to halve because of radioactive decay
* The Geiger counter count rate from a sample to fall to half its starting level.

The half live of the radioactive element Caesium-134 is 2 hours, so that starting with a Geiger counter reading of 64 clicks a second for a sample of Caesium-134.

* 2 hours later it will be 32 clicks per second
* 4 hours after the start it will be 16 clicks per second
* 8 hours after the start it will be 8 clicks per second
* And so on …

Use the information above to calculate how many clicks for these times after starting.

1. 32 hours
2. *n* hours
3. 3 hours.

### Activity E

**Pollution**

There is a spillage of a chemical into a lake.

The average concentration of the chemical in the lake is now 10 times the recommended level.

The concentration of chemical will be expected to decrease gradually due to natural changes, such as fresh water entering the lake (as rainfall) and water/chemical mixture flowing out of the lake (in rivers).

If it is predicted that the chemical concentration will decrease by 25% every month from its concentration at the beginning of that month.

How long will it take for the lake to return to the recommended level of concentration?

**Cloudy water**

The pollution in a lake makes the water cloudy.

This is an important consideration for scuba-divers, who always dive in pairs and must maintain visual contact for safety reasons.

The intensity of light is reduced by 10% for each 20 cm of water it travels through.

At what depth in the lake will the intensity of light be just 1% of that on the surface?