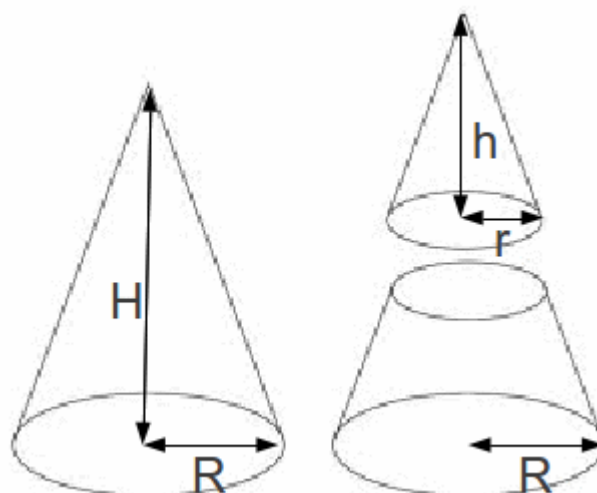


Volumes of Frustums

A frustum is the part of a pyramid or cone lying between parallel planes cutting it. If one of the planes is the base and the other is horizontal then the result is the shape shown below right (for a cone).



The smaller cone top right and the original left are similar shapes, so $\frac{r}{h} = \frac{R}{H}$ (1)

The volume of the original is $V = \frac{1}{3}\pi R^2 H$ and the volume of the smaller cone is $v = \frac{1}{3}\pi r^2 h$. The difference between these two is the volume of the frustum (above bottom right),

$$V_{\text{Frustum}} = V - v = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(R^2 h - r^2 h) \quad (2)$$

Rearranging (1) to give $r = \frac{R}{H}h$ and substituting into (2) gives

$$V_{\text{FRUSTUM}} = \frac{1}{3}\pi(R^2 H - r^2 h) = \frac{1}{3}\pi\left(R^2 H - \left(\frac{R}{H}h\right)^2 h\right) = \frac{1}{3}\pi\left(R^2 H - \frac{R^2 h^3}{H^2}\right) = \frac{1}{3}\pi R^2 H\left(1 - \frac{h^3}{H^3}\right)$$

If $\frac{h}{H} = \frac{1}{2}$ then $V_{\text{FRUSTUM}} = \frac{1}{3}\pi R^2 H\left(1 - \frac{1}{8}\right) = \frac{7}{24}\pi R^2 H$

The volume of the frustum is seven eighths the volume of the cone.