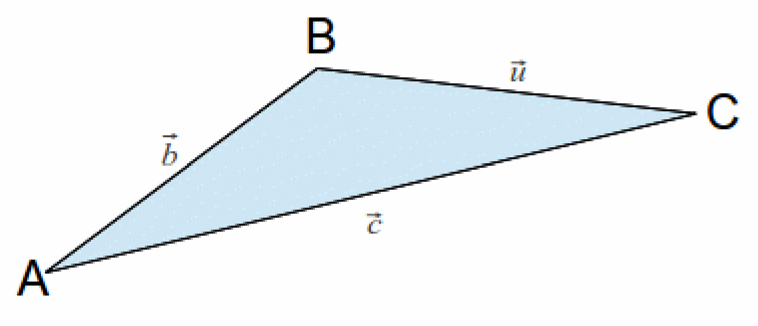
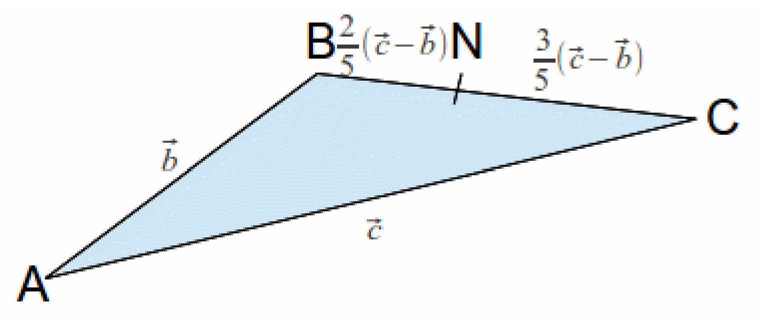
Vectors 2

Vectors can be added in an obvious way. In the diagram below $\vec{b}$ goes from A to B and $\vec{u}$ goes from B to $C.$



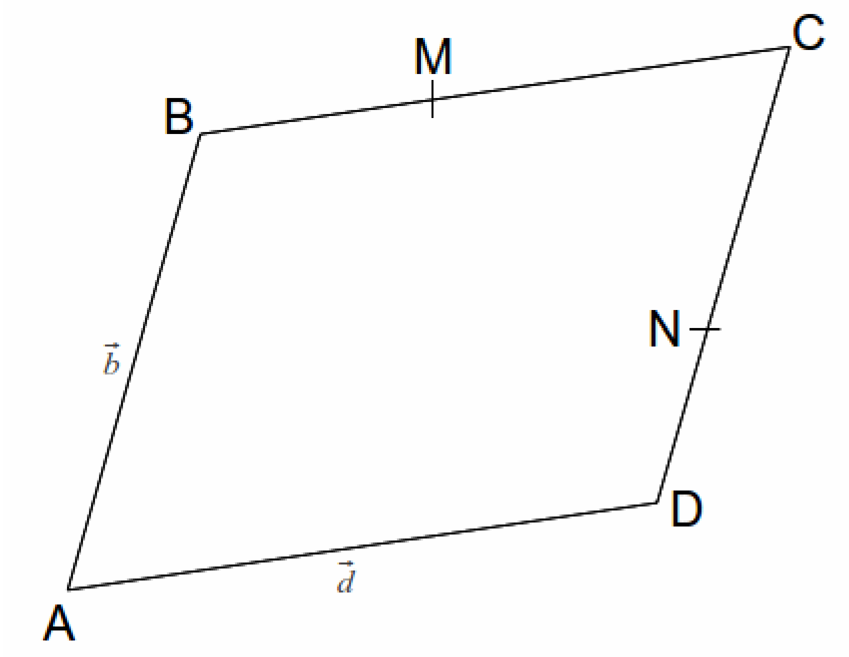
The vector $\vec{c}$ goes from A to $C.$ We can write $\vec{AB}+\vec{BC}=\vec{AC}$ or $\vec{b}+\vec{u}=\vec{c}$ .Importantly, $\vec{u}=-\vec{b}+\vec{c}=\vec{c}-\vec{b}$ so $\vec{BC}=\vec{c}-\vec{b}.$

If BC is split in some ratio by some point N, then we can find $\vec{BN}$ using this ratio. Suppose N splits BC in the ratio 2:3, then $BN:NC$ is in the ratio 2/3 so that $\vec{BN}=\frac{2}{5}\vec{BC}=\frac{2}{5}(\vec{c}-\vec{b})$ and $\vec{NC}=\frac{3}{5}\vec{BC}=\frac{3}{5}(\vec{c}-\vec{b}).$ We can label the triangle as below.



Then $\vec{AN}=\vec{AB}+\vec{BN}=\vec{b}+\frac{2}{5}(\vec{c}-\vec{b})=\frac{1}{5}(5\vec{b}+2\vec{c}-2\vec{b})=\frac{1}{5}(3\vec{b}-2\vec{c}).$

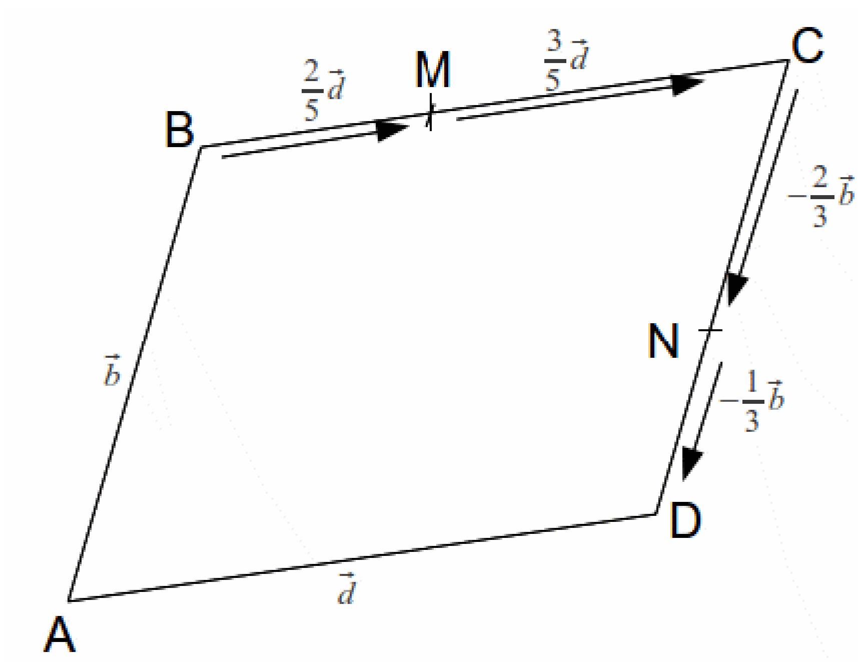
For a slightly more complicated example, consider the parallelogram below, with $\vec{AB}=\vec{b}$ and $\vec{AD}=\vec{d}.$ M splits BC in the ratio 1:2 and N splits CD in the ratio 3:2. Find $\vec{MN}$.



Because ABCD is a parallelogram,

1. $\vec{BC}=\vec{AD}=\vec{d}$ so using the first ratio in the question $\vec{BM}=\frac{2}{5}\vec{BC}=\vec{d}$ and $\vec{MC}=\frac{3}{5}\vec{BC}=\frac{3}{5}\vec{d}$.

2. $\vec{CD}=\vec{BA}=-bec\ b$ so using the second ratio in the question $\vec{CN}=\frac{2}{3}\vec{CD}=\frac{2}{3}(-\vec{b})=-\frac{2}{3}\vec{b}$. and $\vec{ND}=\frac{1}{3}\vec{CD}=-\frac{1}{3}\vec{b}$.



Then $\vec{MN}=\vec{MC}+\vec{CN}=\frac{3}{5}\vec{d}+(-\frac{2}{3}\vec{b})=\frac{3}{5}\vec{d}-\frac{2}{3}\vec{b}$.