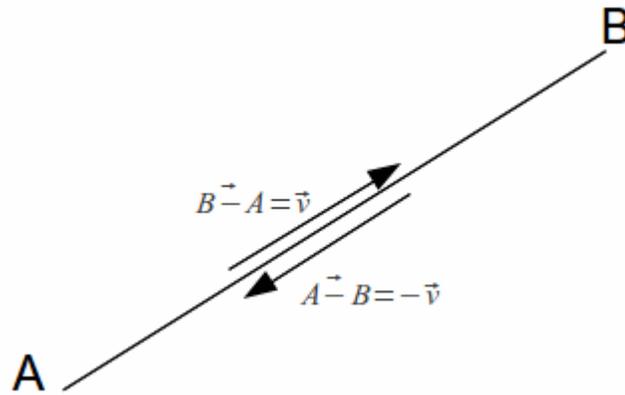


Vectors 1

A vector is the difference between two points. If two points are A and B then we can write the difference $B - A$ (when written in a column between brackets) as $\vec{B - A} = \vec{v}$. The arrow above $B - A$ means that we are going from the point A to the point B . If we swap B and A , then we are going from B to A and write $\vec{A - B} = -\vec{v}$. This is shown below.

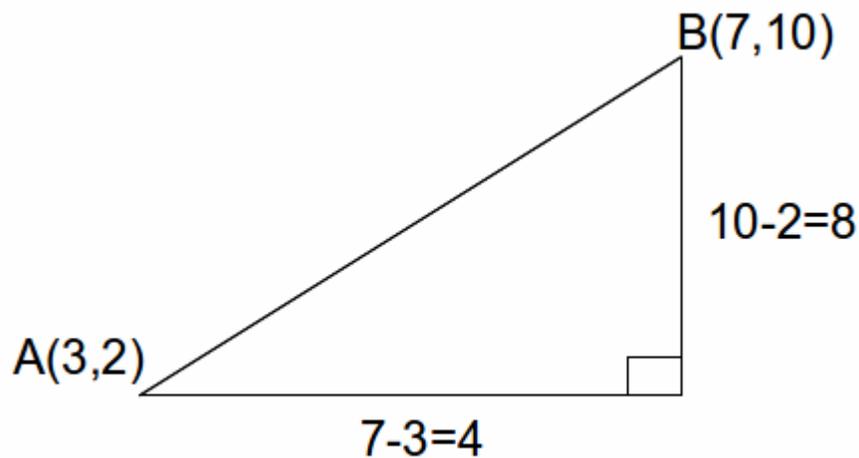


We can also write points as pairs of coordinates and vectors as one number above another. If $A = (3, 2)$ and $B = (7, 10)$ then

$$\vec{B - A} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}.$$

Note that the top number when a vector is written as a column vector is the distance moved in the x direction and the bottom number is the distance moved in the y direction.

The magnitude of a vector is its length or the distance from end to end. We can find the magnitude of a vector written in column form using Pythagoras theorem. For A and B as above, to get from A to B we must go 4 in the x - direction and 8 in the y - direction. These are at right angles as shown below.



Then the length of $\vec{v} = AB = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}$.

We can add and multiply vectors in the natural way so that if $\vec{u} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$ then $\vec{v} + \vec{u} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} + \begin{pmatrix} 8 \\ 1 \end{pmatrix} = \begin{pmatrix} 4+8 \\ 8+1 \end{pmatrix} = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$ and

$$5\vec{v} = 5 \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 5*4 \\ 5*8 \end{pmatrix} = \begin{pmatrix} 20 \\ 40 \end{pmatrix}.$$