

Trial and Improvement

Sometimes it happens that we can't factorise an expression or even use the quadratic formula to solve it, for example $f(x) = x^3 - 2x - 2$. If we try and solve $x^3 - 2x - 2 = 1$, which is equivalent to $x^3 - 2x - 3 = 0$ we cannot do it by these methods. However if we know that the answer is in a certain range we can keep making educated guesses until we get the answer that we know is correct say to one decimal place.

We want to solve $f(x) = x^3 - 2x - 2 = 1$.

We can guess a value of x that satisfies this equation, say $x = 1$:

$f(1) = 1^3 - 2 \cdot 1 - 2 = -3 < 1$ This is too small so x is probably bigger. We can "improve" our guess to $x = 2$.

$f(2) = 2^3 - 2 \cdot 2 - 2 = 2 > 1$ This is too small so now we can conclude the true value of x that satisfies the equation is between 1 and 2. We can draw up the table:

x	x^3	$-2x$	-2	$x^3 - 2x - 2$	Too Big TB Too Small TS
1	1	-2	-2	-3	TS
2	8	-4	-2	2	TB
1.5	3.375	-3	-2	-1.125	TS
1.7	4.913	-3.4	-2	-0.487	TS
1.9	6.859	-3.8	-2	1.059	TB
1.8	5.832	-3.6	-2	0.232	TS
1.85	6.331	-3.7	-2	0.632	TS

The procedure is to use the answer from your last guess to try and get a better value for x . If you guess a value for x and the answer is too small, increase the size of your guess for x . If you guess gives an answer that is too big, guess a smaller value for x . Eventually you will find as we did here with 1.8 and 1.9 that one is too small and one is too big but we can't get any closer by guessing values of x to 1 decimal place.

We have to choose between 1.8 and 1.9, and we do this by trying $x=1.85$. This gave an answer that was too small, so we take the bigger value for x , and to 1 decimal place the solution to the equation $x^3 - 2x - 2 = 1$ is $x=1.9$. If our answer for $x=1.85$ had turned out to be too big, we would have chosen the smaller value for x .