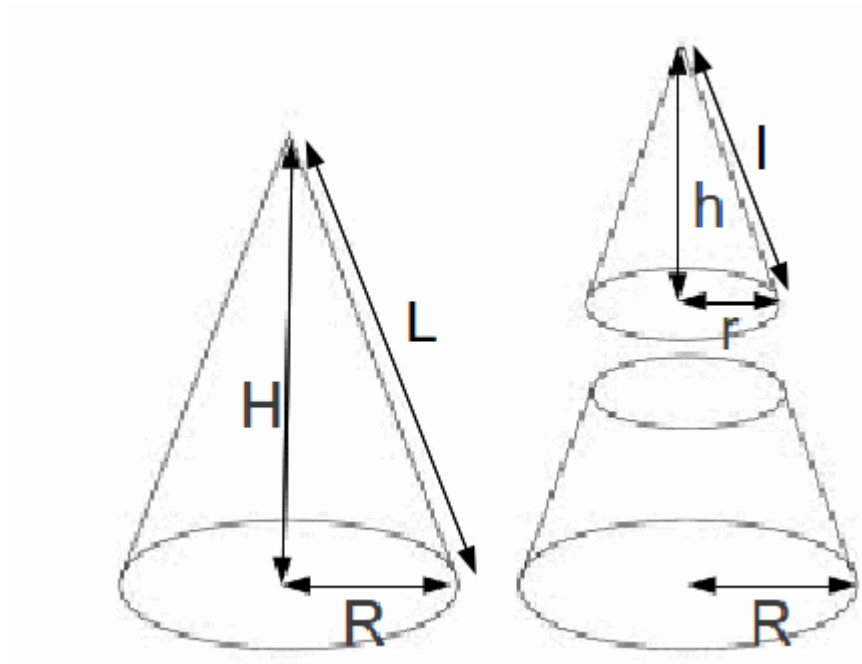


## Surface Areas of Frustums

A frustum is that part of a pyramid or cone that lies between two parallel planes cutting it. If one of the planes is the base and the other is horizontal then the result is the shape shown below right (for a cone).



The smaller cone top right and the original cone left are similar shapes, so  $\frac{r}{h} = \frac{R}{H}$  (1)

The curved surface area of the original cone is  $A = \pi R L$  and the curved surface area of the smaller cone is  $a = \pi r l$ .

The difference between these two is the curved surface area of the frustum

$$A_{\text{CURVED SURFACE AREA FRUSTUM}} = \pi(RL - rl) \quad (\text{above bottom right}), (2)$$

Rearranging (1) to give  $r = \frac{R}{H}h$  and substituting into (2) gives

$$A_{\text{CURVED SURFACE AREA FRUSTUM}} = \pi\left(RL - \left(\frac{R}{H}h\right)l\right)$$

Add on the top and bottom to give

$$A_{\text{SURFACE AREA FRUSTUM}} = \pi\left(RL - \left(\frac{R}{H}h\right)l\right) + \pi r^2 + \pi R^2$$

If  $\frac{h}{H} = \frac{1}{2}$  then  $\frac{r}{R} = \frac{1}{2}$  and  $\frac{l}{L} = \frac{1}{2}$  so  $r = \frac{R}{2}$  and  $l = \frac{L}{2}$ .

$$A_{\text{SURFACE AREA FRUSTUM}} = \pi\left((2r)(2l) - \frac{(2r)}{H}\left(\frac{H}{2}\right)l + \pi r^2 + \pi(2r)^2\right)$$

$$= \pi (4lr - lr + r^2 + 4r^2) = \pi r (3l + 5r)$$