Simple Proofs

Simple proofs often involve simple manipulation of formulae or expressions. Proving an identity is often quite simple, and often shows several examples as an aid. For example

The sum of any four consecutive numbers is equal to the product of the largest two minus the product of the smallest two.

$1+2+3+4=10$ and $4*3-1*2=10$

$2+3+4+5=14$ and $5*4-2*3=4=14$

$3+4+5+6=18$ and $6*5-3*4=18$

A general proof however requires algebra, and the choice of four general consecutive numbers, $n, n+1, n+2, n+3.$

$n+(n+1)+(n+2)+(n+3)=4\mathrm{n}+6$ and $(n+3)*(n+2)-n(n+1)=(n^2+5\mathrm{n}+6)-(n^2+n)=4\mathrm{n}+6$

These expressions are the same so the proof is complete.

Abstract proofs require more though. Suppose we want to prove that the product of consecutive integers is even.

$1*2=2$

$2*3=6$

$3*4=12$

All these are even, but as before, for a general proof we require general consecutive numbers, n and $n+1.$

If n is odd then $n+1$ is even and so $n*(n+1)$ is even.

If n is even then multiplying by any number is even, so $n*(n+1)$ is even.

Often it is required to disprove a statement by finding a counterexample.

The expression n^2+n+11 returns a prime number for $n=1, 2, 3..5...$ right up to $n=9.$

$1^2+1+11=13$

$2^2+2+11=17$

3^2+3+23 etc

For $n=10, 10^{+10}+11=121=11*11$

A more obvious number to choose is $n=11,$ since if $n=11, n^2, n$ and 11 are all divisible by 11. In fact, no formula of the form n^2+n+k generates only prime number. If you choose $n=k,$ you will always get a composite (non – prime) number.