

Sharing Money Equally

Suppose two people Amy and Beth start with an a total of £100. Amy gives one third of her money to Beth and Beth gives one third of the money she now has to Amy. As a result, Amy and Beth now have the same amount of money each. How much did each start with?

Let Amy start with £A and let Beth start with £B. The sequence of transactions is summarised in the table.

| Transaction | Amy | Beth |
|--|---|---------------------------------|
| | A | B |
| Any gives one third of her money to Beth | $\frac{2A}{3}$ | $B + \frac{A}{3}$ |
| Beth gives one third of her money to Amy | $\frac{2A}{3} + \frac{1}{3}(B + \frac{A}{3})$ | $\frac{2A}{3}(B + \frac{A}{3})$ |

The amounts are now equal so

$$\frac{2A}{3} + \frac{1}{3}(B + \frac{A}{3}) = \frac{2}{3}(B + \frac{A}{3})$$

$$6A + 3B + A = 6B + 2A$$

$$5A = 3B \rightarrow 5A - 3B = 0$$

(1)
Obviously

$$A + B = 100$$

(2)
(1)+ 3(2) gives

$$8A = 300 \rightarrow A = 37.5$$

Processing math: 100%

$$B = 100 - A = 100 - 37.5 = 62.5$$