Prime Numbers

A number is a prime number if the only numbers which divide it are 1 AND the number itself. This means that every prime number has two numbers that divide it. 1 is not a prime number cos the only number that divides 1 is 1.

The prime numbers less than 100 are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

A number which is not prime is composite. All composite numbers can be written as a product of smaller numbers bigger than 1. For example, 20=4*5 so 20 is composite.

Prime numbers have several important properties.

There is no rule for finding all the prime number. A rule here means a formula or a method. For example, the expression $f(n)=n^2+n$ produces the first prime number 2 when $n=1,$ but does not produce the second prime number for any integer value of $n.$

There is no rule for finding only prime numbers. For example, if $f(n)=n^2+n+1,$ then not all values of n which are substituted into this expression will produce prime numbers.

If $n=1,$ then $f(1)=1^2+1+1=3$ which is prime

If $n=2,$ then $f(2)=2^2+2+1=7$ which is prime

If $n=3,$ then $f(3)=3^2+3+1=13$ which is prime

If $n=4,$ then $f(4)=4^2+4+1=21$ which is not prime since 21=3*7.

There is only one even prime number, 2. Every even number bigger than 2 is a composite number, so can be written as a product of prime numbers.

When checking whether or not a number is prime, we have only to check if the number is divisible by prime numbers up to the biggest whole number less than the square root of that number. If it is not divisible by any of these numbers then it is prime. For example:

Is 31 prime?

The biggest whole number less than the square root of 31 is 5, and the prime numbers less than or equal to 5 are 3 and 5. 31 is not divisible by 3 and 5 so 31 is prime.

There are an infinite number of prime numbers. This was proved by the ancient Greeks, one of the first abstract and rigorous proofs in maths. The proof goes something like:

Suppose there are only two prime numbers 3 and 5. Then every number can be written as a product of 3's and 5's. Consider 3*5+1=16. This is not divisible by 3 and 5, so must capable of being written as a product of prime numbers (not 3's or 5's. In fact 16=2*2*2*2 and 2 is prime). So 3 and 5 are not the only prime numbers and there must be some other. This argument can be extended, proving there are an infinite number of prime numbers.