

## Lengths, Area and Volumes

Imagine two cubes, one with sides of length 4cm and one with sides of length 8cm. The ratio of these lengths is 4 : 8 (= 1 : 2).

Since these are cubes, each face has base 4 and 8 respectively, and height 4 and 8 respectively. The area of a face of the first is  $16\text{cm}^2$  and the area of a face of the second is  $64\text{cm}^2$ . The ratio of these areas is 16 : 64 (= 1 : 4).

The volume of the first is  $64\text{cm}^3$  and the volume of the second is  $512\text{cm}^3$ . In general, if the ratio of two lengths (of similar shapes) is  $a : b$ , the ratio of their areas is  $a^2 : b^2$ . The ratio of their volumes is  $a^3 : b^3$ .

The ratio of the length of a mm to a cm is 1:10 (there are 10 mm in a cm). The ratio of their areas expressed in these units (i.e.  $\text{mm}^2$  to  $\text{cm}^2$ ) is  $10^2:1$  (there are  $100\text{mm}^2$  in a  $\text{cm}^2$ ) and the ratio of their volumes ( $\text{mm}^3$  to  $\text{cm}^3$ ) is  $10^3:1$  (there are  $1000\text{mm}^3$  in a  $\text{cm}^3$ ).

## Dimensions

Lines have one dimension, areas have two dimensions and volumes have three. We can see this from their respective units: m,  $\text{m}^2$  and  $\text{m}^3$  respectively. We obtain areas by multiplying two lengths together and volumes by multiplying three lengths together. If you use also the fact that you can only add lengths to lengths, areas to areas and volumes to volumes, it is quite easy to pick out those expressions which identify lengths, areas or volumes, or represent nothing at all.

## Examples

The letters  $r$ ,  $l$ ,  $a$  and  $b$  represent lengths.

$\pi r^2 l$  is a volume since  $\pi$  is a number with no units and  $r \times r \times l$  has the units  $\text{m}^3$ .

$2\pi r^2$  is an area. Ignore  $2\pi$  and  $r^2$  has units  $\text{m}^2$ .

$4\pi r^3$  is a volume. Ignore  $4\pi$  and  $r^3$  has units  $\text{m}^3$ .

$abrl$  is not a length, area or volume since the units are  $\text{m}^4$ .

$\frac{abl}{r}$  is an area since the units are  $\text{m}^3/\text{m}=\text{m}^2$ .

$3(a^2 + b^2)r$  is a volume.

$\pi rl$  is an area.