

# Inverting Functions

A function  $f$  allows us to find  $f(x)$  for any value of  $x$ . We just substitute the value for  $x$  into  $f(x)$ .

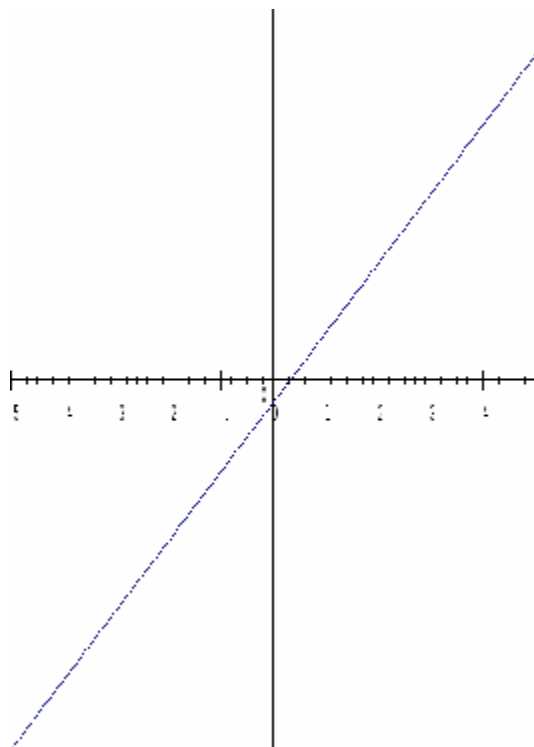
If  $f(x) = x^2 + 3$ , then we find  $f(x)$  when  $x = 1$  by calculating  $f(1) = 1^2 + 3 = 4$ .

Inverting a function allows us to find  $x$  for any value of  $f(x)$  if this is possible.

The Procedure is:

1. Write  $y = f(x)$ .
2. Rearrange the equation to make  $x$  the subject.
3. Swap  $x$  and  $y$  over.
4. Replace  $y$  by  $f^{-1}(x)$
5. The graph of  $f^{-1}(x)$  may be sketched by reflecting the graph of  $y = f(x)$  in the line  $y = x$ .

Example: The graph of  $f(x) = \frac{3x-1}{2}$  is shown below.



$$y = \frac{3x-1}{2}$$

Make  $x$  the subject:  $y = \frac{3x-1}{2} \rightarrow 2y = 3x-1 \rightarrow 2y+1 = 3x \rightarrow \frac{2y+1}{3} = x.$

Swap  $x$  and  $y$ :  $\frac{2x+1}{3} = y$

Replace  $y$  by  $f^{-1}(x)$ :  $f^{-1}(x) = \frac{2x+1}{3}.$

