

Arithmetic Sequences

An arithmetic sequence is any sequence where a fixed number is added to a term to get the next term.

3, 7, 11, 15, 19

is an arithmetic sequence with 4 being added to each term to get the next term.

Given any arithmetic sequence we can find an expression for the n th term. If d is the number that is added each time (called the common difference) and a is the first term, then the n th term is $a_n = a + (n - 1)d$.

For the sequence above, $a = 3$, $d = 4$.

Hence $a_n = 3 + (n - 1) \times 4 = 4n - 1$

We can also find a formula for the sum S_n of the first n terms.

$$S_n = a + (a + d) + \dots + (a + (n - 2)d) + (a + (n - 1)d)$$

Writing this sum backwards gives

$$S_n = (a + (n - 1)d) + (a + (n - 2)d) + \dots + (a + d) + a$$

Now adding these two sums gives

$$2S_n = \underbrace{(a + (n - 1)d) + (a + (n - 1)d) + \dots + (a + (n - 1)d) + (a + (n - 1)d)}_{n \text{ terms}}$$

$$= n(2a + (n - 1)d)$$

$$\text{Hence } S_n = \frac{n}{2}(2a + (n - 1)d)$$

For the sequence above the sum of the first 20 terms is $S_{20} = \frac{20}{2} \times 3 + (20 - 1) \times 4 = 820$