

Advanced Indices

Advanced indices questions may take several forms. One of these may require you to compare the magnitude of different expressions. For example:

If $x > 1$, arrange in order of magnitude

$$1, \sqrt{x}, x^3, \frac{1}{x}, \frac{1}{x^2} \cdot x$$

If $x > 1$ then x^k increases with increasing k , so a bigger value of k means a bigger value of x^k .

Note that $1 = x^0, \sqrt{x} = x^{\frac{1}{2}}, \frac{1}{x} = x^{-1}, \frac{1}{x^n} = x^{-n}$ and $x = x^1$. (1)

Since $-2 < -1 < 0 < \frac{1}{2} < 3$ we have $\frac{1}{x^2} < \frac{1}{x} < 1 < \sqrt{x} < x^3$.

If $0 < x < 1$, arrange in order of magnitude

$$1, \sqrt[3]{x}, x^4, \frac{1}{\sqrt{x}}, \frac{1}{x^3} \cdot x$$

If $0 < x < 1$ then x^k decreases with increasing k , so a bigger value of k means a smaller value of x^k .

Using (1) and that $\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}, \sqrt[3]{x} = x^{\frac{1}{3}}$ and

Since $-3 < -\frac{1}{2} < 0 < 1 < \frac{1}{3} < 4$ we have $x^4 < x < \sqrt[3]{x} < 1 < \frac{1}{\sqrt{x}} < \frac{1}{x^3}$.