International Baccalaureate

Extended essay cover

SAMPLE B

Diploma Programme subject in which this extended essay is registered: Mathematics

(For an extended essay in the area of languages, state the language and whether it is group 1 or group 2.)

Title of the extended essay: Could the Green Arrow's Bow shoot Arrows of more than One pound using mathematics?

Candidate's declaration

If this declaration is not signed by the candidate the extended essay will not be assessed.

The extended essay I am submitting is my own work (apart from guidance allowed by the International Baccalaureate).

I have acknowledged each use of the words, graphics or ideas of another person, whether written, oral or visual.

I am aware that the word limit for all extended essays is 4000 words and that examiners are not required to read beyond this limit.

This is the final version of my extended essay.

Candidate's signature: ______________________ Date: 2/2/09

IB Cardiff use only: A: ________ B: ✓

International Baccalaureate, Peterson House,
Malthouse Avenue, Cardiff Gate, Cardiff CF23 8GL, Wales, United Kingdom

Supervisor's report

The supervisor must complete the report below and then give the final version of the extended essay, with this cover attached, to the Diploma Programme coordinator. The supervisor must sign this report; otherwise the extended essay will not be assessed and may be returned to the school.

Name of supervisor (CAPITAL letters) ______________________

Comments

Please comment, as appropriate, on the candidate's performance, the context in which the candidate undertook the research for the extended essay, any difficulties encountered and how these were overcome (see page 13 of the extended essay guide). The concluding interview (viva voce) may provide useful information. These comments can help the examiner award a level for criterion K (holistic judgment). Do not comment on any adverse personal circumstances that may have affected the candidate. If the amount of time spent with the candidate was zero, you must explain this, in particular how it was then possible to authenticate the essay as the candidate's own work. You may attach an additional sheet if there is insufficient space here.

The candidate was very enthusiastic about this topic and through our numerous meetings this paper went through enormeous revisions. Although the paper is still riddled with many issues it was a tremendous improvement upon earlier renditions.

I have read the final version of the extended essay that will be submitted to the examiner.

To the best of my knowledge, the extended essay is the authentic work of the candidate.

I spent | 15 | hours with the candidate discussing the progress of the extended essay.

Supervisor's signature: ______________________ Date: 2-2-09

An unusual essay with apparent personal interest and contribution, some Math conte[nt] and original "research" question. The metho[d] of investigation however is not [illegible] There is so[me] very clear.

obvious/and not very sophisticated use of geometry and [illegible] in the modeling process, mention of dif. equations, but overall it is not clear [illegible]
There is simulation or calculation of a model. I am accepting that the candidate unders[tands] (giving the benefit of doubt) however presentati[on] is more to the poor side

WORLD SCHOOL ®

IB Diploma

Extended Essay

Class of 2009

Extended Essay
Mathematics

Could the Green Arrow's Bow Shoot Arrows that weigh more than One Pound?

By

Session: May 2009
Candidate Number:
Word Count: 2,739

Thanks to:

Abstract

Using the dimensions of the Green Arrow's bows given by the writer Eliot R. Brown, I will try to test the validity of the Emerald Archer shooting an arrow with a boxing glove tip, by determining how far his bow could shoot an arrow of a range of sizes, and concluding if the distance shot for that weight is strong enough to cause any damage to anything. In this I found that only if the bow limbs bent at the top of the riser and the curve allowed it to fit into a stretch of about 7.5 and the spring force angle was at 0 could the bow create enough force to shoot an arrow that could weigh enough to have a boxing glove on the tip.

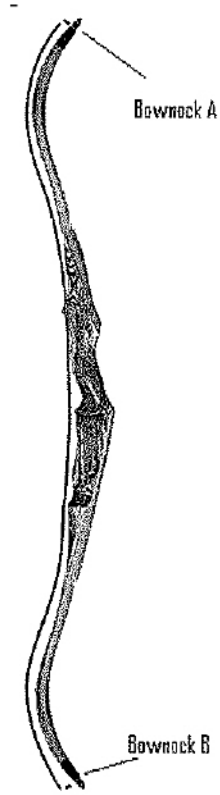
Word Count: 127

Table of Contents

Introduction

My favorite thing about mathematics is also my favorite thing about comic books. With both, one can escape and become totally absorbed in finding the solution to a problem. In mathematics that problem may have numerous solutions, and on the same token, comic books stories often have numerous dimensions for unlimited outcomes and solutions to the hero's problem. The Green Arrow comics, particularly the Quiver series, are of special interest to me. The Green Arrow, aka Oliver Queen, is most well known for his quiver of novelty arrows, including a boxing glove arrow, an explosive arrow and a grappling hook arrow. The completely fictitious nature of his trick arrows actually flying from his bow posed a ponderous question, could these arrows actually be shot from his bow? Then, the Origins comic fell into my hands.

The comic, issue 1 of Green Arrow released in December of 2002, Secret Files & Origins, like most comic books, included multiple writers, but only one provided these details on the Archer's weapons. The writer, Eliot R. Brown, gave the measurements of a 46 inch bow of the "recurve type" and with a 125 lb "pull," he also disclosed that Oliver Queen, the Green Arrow, is 5'11". After a fair amount of investigation, I came to the understanding that it was not "recurve type" but a recurve bow, and that the "pull" referred to the draw weight. I was then able to use those measurements and other characteristics of recurve bows, such as the knowledge that their bow string is four inches shorter than the bow and that the length of the bow is measured along the belly, or inward side, and measures along the two limbs and the riser. The focus of this essay is to find the best possible draw angles, and to use the draw weight and the draw length, in combination with different bow and arrow masses, to determine the furthest distances of which the bow can fire different arrow weights.

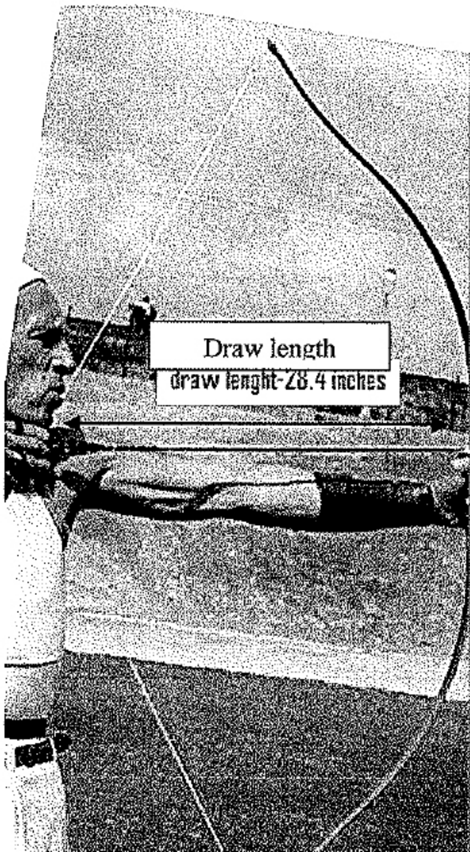


Determining the Draw Length

The bow length is found by measuring from bow nock to bow nock on the belly side of the bow. When stringing a bow, the rule of thumb is to acquire a string 4 inches shorter than the bow (Besherse). The bow length given by Eliot R. Brown is 46 inches, which indicates that the bowstring should be 42 inches long. The bow curve, or the curve in the bow limb, increases the amount of spring force exerted when the bowstring is pulled back, and thus how much force gets exerted into the arrow.

To determine the draw length of a person, the person should measure the distance from the tip of one middle finger to the tip of the opposite middle finger, with their arms spread, perpendicular to their body and parallel to the floor, then divide this value by 2.5. This figure reflects a distance 20% shorter than the entire distance of a persons arm. This is the standard length a person holds a bow from their body while holding the bowstring. Since this

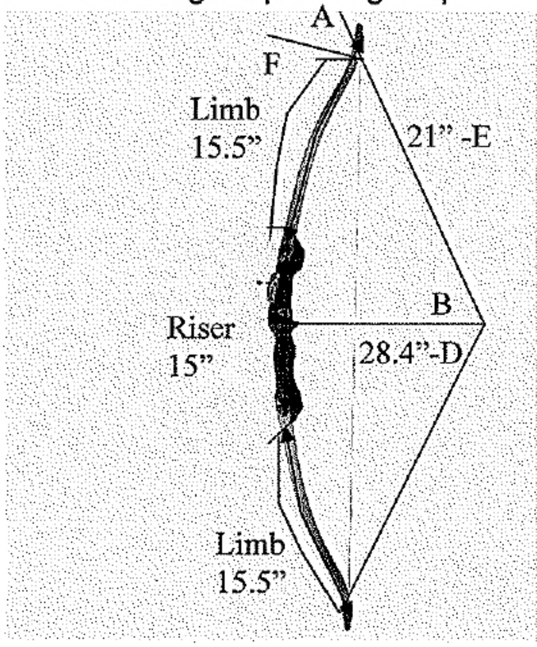
Source?



distance is approximately the height of the individual, I will use Oliver Queen's height to determine his draw length:

$$5\,foot\,11\,inches = 71\,inches$$
$$71 \big/ 2.5 = 28.4\,inches$$

The reason the height can be used in place of the finger tip to finger tip measurement with minimal variation is a result of a biological symmetry. Biological symmetry is one of the wonders of the human body where unrelated body parts have similar, if not almost identical, measurements. Another such example is the similarity between the length of a person's foot and the length of their forearm.



Knowing the draw length of the Green Arrow then gives us the values of D, the draw length and of E, half the bowstring, but still leaves no specific value for any of the other angles.

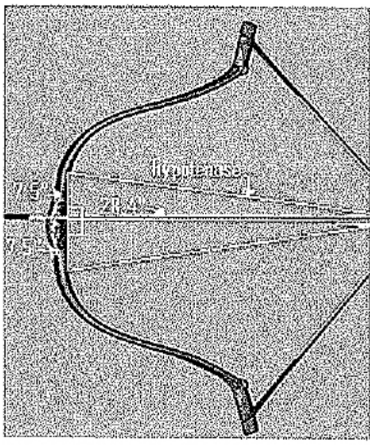
However, the length of the limbs can be predicted. Considering the curve of the bow limbs, the measure of half the bow length, 23 inches, is too large. Also, the riser, will remain the same length. Since the material bends, but would still measure to be 15.5 inches if measured along the wood of the limb at any draw length, the length of the bow equals 46 inches. The length of the riser, we'll assume to be a little less than $\frac{1}{3}$, since the majority of a bow is comprised of the two bow limbs. This measurement however can be any figure provided it leaves enough length for the bow limbs to be larger than the riser and the total length of the two bow limbs and the riser do not exceed 46 inches. The total length, 15 $\frac{1}{3}$ rounded to 15 inches, leaves 31 inches for the limbs. Since the two limbs of a recurve bow are equal in length, the length each limb is then 15.5 inches. The 15 inches will remain constant since the riser does not bend, unlike the limbs. The remaining 15.5 is only relevant in its effect on angle B, which is found proportional to the distance from the top of the riser to nearest nock in a straight line.

The Distances:

The distance that the arrows fly is determined by a series of equations. First, the velocity of the arrow was determined using the formula $Velocity = \sqrt{\frac{2*energy}{mass}}$. This velocity was then entered into an equation which yields the vertical displacement based on the angle that the arrow was shot. For these the angle was at cos(30) sin(60). The Velocity times the sin(60) is equal to the vertical displacement. The time in flight is the vertical displacement divided by the pull of gravity $32 fts^{-2}$, times two, since the arrow travels up then down. The time in flight is then multiplied by the horizontal displacement, or the velocity times the cosine of 30. This was done for each trial.

In order to determine the maximum distance an arrow can travel when shot from a bow of these proportions by a man of this height, several calculations were needed.

In my first scenario, I assume that the bow is made of an extremely flexible material to allow for the minimum riser to bow nock distance. Using the following dimensions we can find the length necessary to determine the draw force of the bow. We know that when the arrow is cocked perpendicular to the riser and along the middle of it, that the isosceles triangle at the right is produced. Using the right triangle to find the hypotenuse we use Pythagoreans theorem: $A^2 + B^2 = C^2$

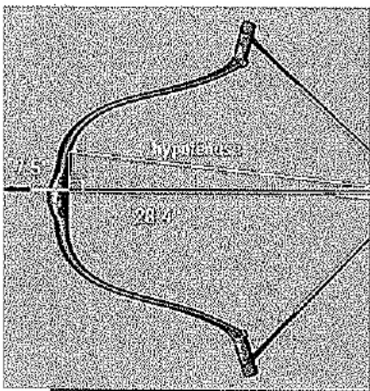


$A = {}^{15}\!/_{2}$

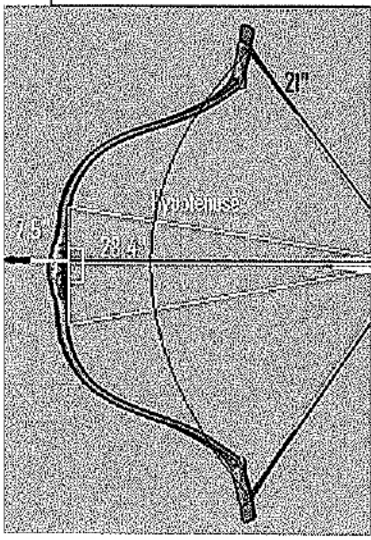
$B = 28.4$

$C = \sqrt{862.81}$

$$A^2 + B^2 = C^2$$
$$(7.5)^2 + (28.4)^2 = C^2$$
$$56.25 + 806.56 = C^2$$
$$862.81 = C^2$$
$$\sqrt{862.81} = \sqrt{C^2}$$
$$29.37362763 inches = C$$



Taiwan of Archery, Bow, Crossb
Arrow Manufacturer, Supplier -

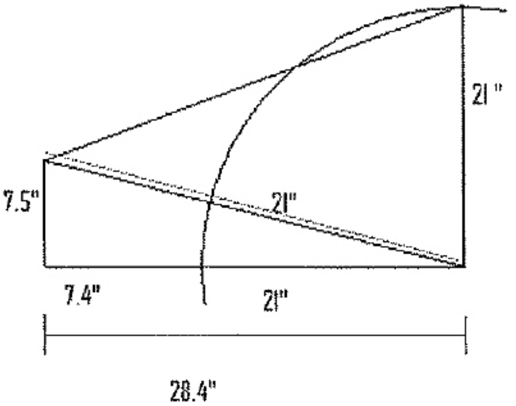


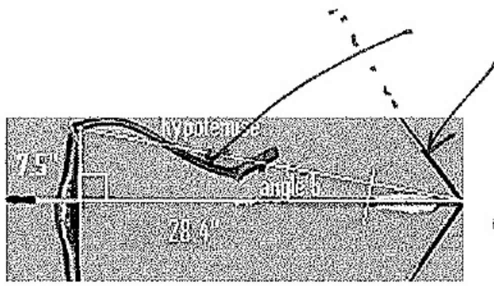
Since the string used does not change length, then if the arrow is perpendicular to the riser, it will hit the middle of the bowstring. The length of 7.5 inches represents the upper half of the riser, the measure of 21 inches represents the length of bow string halved by the arrow, which is shown as the bottom of the image, at 28.4 inches. The hypotenuse of the triangle created by the riser over the arrow, shows the minimum length of the bow string and the upper limb, since the shortest distance between two points is a straight line.

Minimum Riser to Bow Nock Distance

which one?

The length of this hypotenuse is C inches, which shows when the bow limb is bent to make a straight line with the bowstring. The angle between the bow string and the arrow at the minimum is found using the law of sine's, which states that each side is proportional to its angle. FALSE





$$\tan^{-1}\left(\frac{7.5}{28.4}\right) = b$$
$$b = 14.79320394\ ^\circ$$
$$b = .2581901157 \quad radians$$

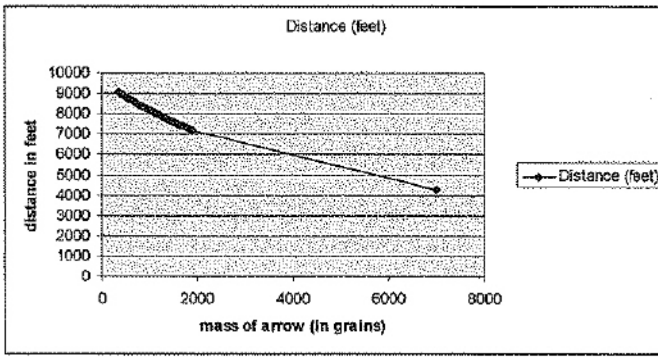
Since the hypotenuse's opposing angle is 90 degrees and the opposing side of the angle produced by the string and the arrow is 7.5 inches, according to the law the angle is approximately 14.79 degrees.

where does this come from?

The draw force is equal to the cosine of angle b, the cosine of the angle between the spring force and the bowstring, and the spring force constant. The draw weight of the bow is 125 foot-pounds, which means that it

grains	Distance (feet)
350	9080.705
400	9005.032
450	8930.61
500	8857.409
550	8785.397
850	8376.774
900	8312.337
950	8248.884
1000	8186.393
1050	8124.841
1100	8064.208
1150	8004.473
1200	7945.617
1250	7887.619
1300	7830.463
1350	7774.128
1650	7452.44
1900	7204.026
7000	4288.111

how is this calc



takes 125 pounds of force to move an object one foot. Since the bown limb and the bowstring create a straight line, the angle between the spring force and the bowstring is 0. Therefore, force of the bow is equal to:

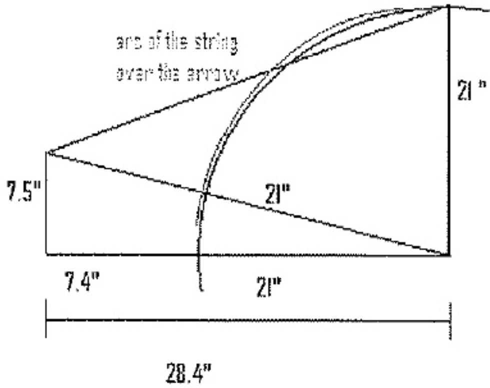
$$Force = 125 \times 2 \times \cos(b)$$
$$Force = 36898.300986\, foot - pounds$$

The total amount of foot pounds stored in the bow is demonstrated above. Using that stored energy in the equation the relationship can be demonstrated as such. Based on this equation, if the bow limb bends to that angle when the bow is drawn to 28.4 inches, the tension in the limb is such that the force will spring an arrow weighing 6 pounds about 1132 feet. Which gives some potential to the boxing glove arrow, which is essentially a boxing glove placed on the tip of an arrow, and for it to have any amount of impact, it would need to have some hefty weight to it. Wind resistance would still be a factoring issue, but if the bow can produce a force of relatively 36898.300986 foot-pounds, it can create enough velocity to throw such an arrow. The issue concerning the bow however, is that no bow limbs could possibly bend that far back, and in doing so, the bow limb would logically break off the riser. The table shows the distances associated with this bend and force based on the grain weight of the arrows. One grain weight is equivalent to $\frac{1}{7000}$ of a pound.

Maximum Riser to Bow Nock Distance:

In my next scenario I will assume an extremely ridged bow that does not bund with its curves, but maintains its length for a maximum riser to bow nock distance.

To use the upper limb at 15.5 inches to find the angle B, would require the curve of the limb to cease being a curve and for the whole limb to be perfectly straight and the arrow to rest at the top of the riser, which would mean the curve comes after the nock of the bow. Thus, the distance from the top of the riser to top of the string cannot exceed 15.5 inches.



Using the equation of the circle created by spinning 21 inch string around the end of the draw length farthest from the belly of the bow, and finding the distance from the top of the riser to the edge of that circle, I can find the maximum length proportional to the angle of the bowstring to the arrow.

X	y	distance
10.5	10.98135	11.06209
11	11.75755	11.7952
11.5	12.46555	12.52624
12	13.1164	13.2493
12.5	13.71824	13.96125
13	14.27725	14.66053
13.5	14.79831	15.34651
14	15.28529	16.01907
14.5	15.74135	16.67842
15	16.16911	17.32494
15.5	16.57076	17.95908

According to the table, the segment of the circle creates a distance from the top of the riser greater than the length of the limb between the points of (13.5, 14.79) and (14, 15.29).

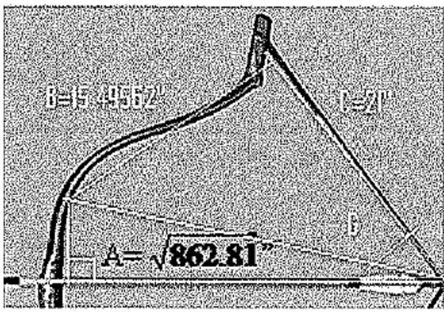
x	y	distance
13.5	14.79831	15.34651
13.6	14.89832	15.48209
13.7	14.997	15.61714
13.8	15.09437	15.75165
13.9	15.19046	15.88563
14	15.28529	16.01907

So, I began to explore further, between 13.5 and 14 for values of x, and after that further into the values of x between 13.6 and 13.7, to get a more accurate account of the distances between the riser and the top of the bow string.

As a matter of personal choice, I will use 15.49562 inches as the maximum, since it still allows for a curve in the bow limb, but is still near enough to the hypothetical straightened limb to be a maximum length.

x	y	distance
13.6	14.89832	15.48209
13.61	14.90825	15.49562
13.62	14.91816	15.50914
13.63	14.92806	15.52266
13.64	14.93795	15.53617
13.65	14.94783	15.54968

Now, using the length of the maximum, 15.49562 inches, and the length of the bow string, 21 inches, and the length of the hypotenuse, to find the angle of G in the diagram:



One equation using the law of cosines can use the three sides to produce the angle G.

$$g^2 = A^2 + C^2 - 2 \times A \times C \times \cos(g)$$

$$15.49562^2 = \sqrt{862.81}^2 + 21^2 - 2 \times \sqrt{862.81} \times 21 \times$$

$$240.1142392 = 862.81 + 441 - 42\sqrt{862.81} \times \cos($$

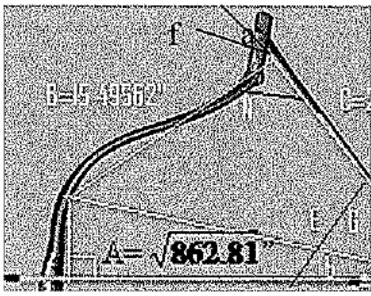
$$240.1142392 - 862.81 - 441 = -42\sqrt{862.81} \times \cos$$

$$\frac{-1063.695761}{-42\sqrt{862.81}} = \cos(g)$$

$$\cos^{-1}\left(\frac{1063.695761}{42\sqrt{862.81}}\right) = g$$

$$G = 30.43492739°$$

The purpose of this is to find the angle between the bowstring and the arrow. Since we know the angle between the hypotenuse and the arrow, we can add that plus the angle of G, and using that angle,



finally begin to find the total energy stored in the bow. The angle between the bowstring and the arrow is equal to:

$G + b = 45.22813133°$ =E

The angle between the spring force and the bowstring is half the angle between the bow limb and the bowstring. This can be determined by first finding the value of N, the angle between the bow limb and the bowstring:

$$862.81 = 15.49562^2 + 21^2 - 2(15.49562)(21)\cos(N)$$
$$N = \cos^{-1}\left(\frac{862.81 - (15.49562^2 + 21^2)}{-2(15.49562)(21)}\right)$$
$$N = \cos^{-1}(-.2791538229)$$
$$N = 106.2113567$$
$$a = N \times \frac{1}{2}$$
$$a = 53.1567836$$

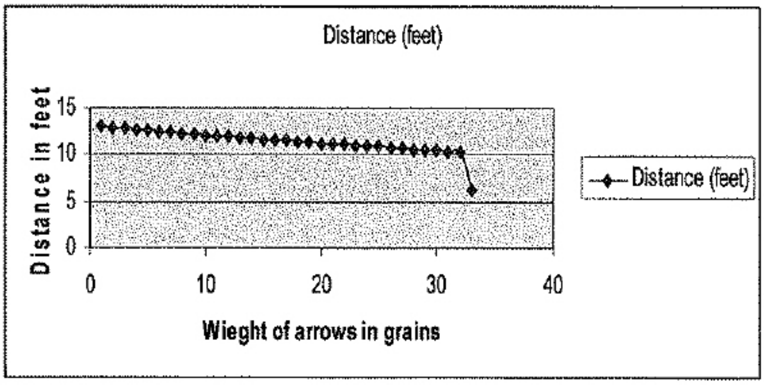
there are two different concepts

The total work, or force, is equal to:

$$Force = 125 \times 2 \times \cos(b + G) \times \cos(a)$$
$$Force = 105.7028954 \textit{ foot - pounds}$$

Given this force, the relationship between the weight of the arrow and the distance it will fly, ignoring wind resistance, is demonstrated as follows.

grains	Distance (feet)
350	13.00679
600	12.48232
900	11.90621
1100	11.5508
1400	11.05577
1800	10.45816
1850	10.38797
1900	10.31872
7000	6.142094



where does this come from?

Median Riser to Bow Nock Distance:

In my final scenario, I will assume a median riser to bow nock distance. A median bow limb distance, between the maximum and the minimum, is found by subtracting the length of the minimum from the length of the maximum distance from the top of the riser to the top of the bowstring. The maximum length is 15.49562 inches from bowstring to riser, and the minimum distance is 8.37363 inches, where the limb is level with the string. To find the distance halfway between the two, you take the distance from one to the other, divide it in half, and add the quotient to the minimum length. The maximum is 7.12199 inches from the minimum. The mid-point of the two distances is where the upper end of the bowstring is 11.934625 inches from the riser. Returning to the cosines, the angle between the arrow and the string is 34.50839362 degrees. Again, if the force angle is equal to half of the angle between the bowstring and the bow limb, the same formula can be used as previously, except using the value of j in place of G, which will yield a different n.

$$B^2 = A^2 + C^2 - 2 \times A \times C \times \cos(b)$$
$$11.934625^2 = \sqrt{862.81}^2 + 21^2 - 2 \times \sqrt{862.81} \times 21$$
$$11.934625^2 = 862.81 + 441 - 42\sqrt{862.81} \times \cos(j)$$
$$11.934625^2 - 862.81 - 441 = -42\sqrt{862.81} \times \cos($$
$$\frac{11.934625^2 - 862.81 - 441}{-42\sqrt{862.81}} = \cos(j)$$
$$\cos^{-1}\left(\frac{11.934625^2 - 862.81 - 441}{42\sqrt{862.81}}\right) = j$$
$$j = 19.71518967$$
$$b + j = 34.50839362°$$

$$862.81 = 11.934625^2 + 21^2 - 2(11.934625)(21)\cos$$
$$N = \cos^{-1}\left(\frac{862.81 - (11.934625^2 + 21^2)}{-2(11.934625)(21)}\right)$$
$$N = 123.8728222$$
$$a = N \times \frac{1}{2}$$
$$a = 61.93641111$$

A is then used in the equation to yield the total force energy of the bow:

$$Force = 125 \times 2 \times \cos(b + j) \times \cos(a)$$
$$Force = 96.91802848\, foot - pounds$$

The relationship between the weight of the arrows and the resulting distance is given as: (See table to right.)

grains	Distance (feet)
350	11.92581
400	11.82642
450	11.72869
500	11.63255
550	11.53798
650	11.35337
700	11.26326
800	11.08727
850	11.00133
950	10.83337
1050	10.67046
1250	10.35891
1350	10.20986
1400	10.13694
1450	10.06504
1500	9.994162
1700	9.720349
1750	9.654224
1800	9.588993
1900	9.46114
7000	5.631631

The values are less than both the max and the min as a result of the angle of A, from which the spring force interacts with the bow.

In Conclusion

The reason that the arrows did not launch impressive distances is a result of the short limbs of the bow. Trying to store enough energy in its relatively small limbs does not allow for enough energy storage to launch an arrow a good distance. Therefore, unless the Green Arrow managed to perfect that spring force arrow at 90 degrees, he would need to use arrows lighter than .05lbs, which would not be able to hit villains with enough force. These findings are rather disheartening.

However, since the Star City exists in DC comics, the Green Arrow's bow and arrows can continue to shoot for justice.

This is basically an EE in physics. The physics appears to be incorrect (and the use of non standard physical units like feet, grains, pounds, etc... contributes to make the computations unverifiab[le] but this is not my problem as an examiner [in] mathematics.

The essay, however, from a mathematical point of view is nearly empty, involving on[ly] elementary geometry and being couched in a very confusing language, lacking crucial diag[rams]. Mathematical results are misquoted (e.g the "sine law") [...] tables are produced from nowhere.

Altogether a poor essay (not surprisingly i[n] view of the poor choice of subject, almost certain[ly] lead to mathematically empty essay)

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Assessment form (for examiner use only)

Candidate session number	0	0	

Assessment criteria		First examiner	maximum	Second examiner
	A research question	2	2	1
	B introduction	1	2	1
	C investigation	3	4	2
	D knowledge and understanding	3	4	2
	E reasoned argument	1	4	2
	F analysis and evaluation	2	4	2
	G use of subject language	1	4	1
	H conclusion	2	2	2
	I formal presentation	3	4	2
	J abstract	2	2	2
	K holistic judgment	3	4	2
	Total out of 36	23		19

(Achievement level)

Name of first examiner: ______________________ Examiner number: ________
(CAPITAL letters)

Name of second examiner: ______________________ Examiner number: ________
(CAPITAL letters)